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Advertising and Entry Deterrence:
An Exploratory Model

by

Richard Schmalensee

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Richard Schmalensee

Sloan School of Management

Massachusetts Institute of Technology

ABSTRACT

In this model, the effects of advertising are infinitely durable, fixed (and sunk) costs give rise to economies of scale, post-entry behavior is non-cooperative, and pre-entry expectations are rational. Despite the obvious resemblance to work on the use of investment in production capacity to deter entry, here the incumbent monopolist never finds it optimal to advertise more if entry is possible than if it is not. This result and other features of this model indicate the dangers of analyzing advertising by analogy with other sorts of investments. Our results make clear the need for more theoretical work on advertising and entry deterrence.

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A basic defect in the classic Bain/Sylos limit-pricing model of entry deterrence stems from its assumption that the potential entrant believes that the established firm would maintain its output constant if entry occurred. The problem is that if entry did occur, it would not generally be rational for the established firm to carry out this threat; thus the threat is not credible.[1] Recent work on entry deterrence has begun to correct this defect by exploring the consequences of ruling out such irrational post-entry behavior and of assuming that potential entrants have rational expectations.[2] A leading result in that literature is that in the presence of economies of scale, an established monopoly facing the threat of entry might find it possible and optimal to deter that entry credibly by investing more in durable production capacity than it would if no such threat were present.[3] Both the durability of the investment and the presence of economies of scale are apparently necessary for this result.

It is generally acknowledged that the effects of advertising persist over time, and it is often argued that there are important economies of scale in advertising.[4] Reasoning by analogy with investment in production capacity, as is commonly done in this context, one might conjecture (as I did initially) that in any plausible model assuming durability and scale economies, it will sometimes be optimal for an established monopolist to over-advertise in order to deter entry credibly.

Baldani and Masson (1981) have recently built a model which is consistent with this conjecture. Their over-advertising result seems a bit forced, however, since under their assumptions a monopolist protected from entry would spend nothing at all on advertising. If the incumbent advertises in advance of entry, it is assumed to create durable "goodwill" that forces the entrant to advertise in order to sell anything; the greater the incumbent's "goodwill," the lower the entrant's sales at all advertising levels. If entry does not occur, the incumbent's "goodwill" is assumed not to affect its demand. The assumed properties of "goodwill" are not related to a model of individual buyer behavior, and post-entry equilibrium is not fully modeled.

This essay presents an alternative model, patterned on those focusing on investment in production capacity and constructed to explore the role of advertising in entry deterrence. Though the assumptions made here have a superficial resemblance to those of Baldani and Masson (1981), our results are quite different. In this model, the effects of advertising are infinitely durable, fixed costs give rise to economies of scale, post-entry behavior is noncooperative, and pre-entry expectations are rational. Yet the incumbent monopoly _never_ finds it optimal to advertise more if entry is possible than if it is not. Optimal entry deterrence always involves advertising _less_ than if the threat of entry were absent. Certain disadvantages of size appear in this model's post-entry equilibria, and it may be better to enter second than first. These results and others presented below make plain the dangers of analyzing investments in advertising largely by analogy with investments in production capacity. They also suggest that the strategic implications of investments in advertising may be highly sensitive to the effects of advertising on buyer behavior and to the nature of post-entry equilibrium.

I. Assumptions

We deal throughout with only two firms, X and Y. Perhaps because of stochastic factors in the product development process, firm X appears on the market first. When firm Y subsequently appears, it may or may not elect to enter. Both firms produce the same product if they produce anything.

There is a continuum of potential buyers with linear individual demand curves. The total mass of these buyers is normalized to be unity for convenience, and the units in which prices and quantities are measured are chosen so that if a fraction θ of potential buyers are aware of the product, their total demand is given by

$$q = \theta(1 - p), \text{ or } p = 1 - (q/\theta), \tag{1}$$

where p is the market price and q is the total quantity demanded. The unit cost of production, γ, is a constant less than one for both firms. None of what follows seems to depend critically on demand linearity or cost constancy.

In order to make any sales, firm X must invest a fixed amount, f, to design leaflets and prepare to print them. This cost is forever sunk.[6] It then prints leaflets and sends them at random to potential buyers, exactly as in Butters (1977). There is a constant printing and distribution cost per leaflet sent. Buyers who receive one or more leaflets are aware forever of X's location and the attributes of its product. Those who receive no leaflets never learn about this product. The effects of this introductory advertising are thus infinitely durable. Butters (1977) notes that if L leaflets are sent at random to B buyers, and both L and B are large, then the fraction of buyers informed is approximately equal to $[1 - \exp(-L/B)]$. Treating this fraction as exact, and letting a equal B times the printing and distribution cost of a single leaflet, we obtain the cost of informing a fraction x of potential buyers:

$$c(x) = f - a\ln(1 - x). \tag{2}$$

We will use x for the fraction informed by firm X and y for the fraction informed by firm Y, which faces the same advertising cost function. Note that (2) implies a U-shaped average cost function for real advertising, x. Much of what follows holds for all cost functions of this sort.

We begin by analyzing firm X's behavior if subsequent entry Y is impossible. Section III describes post-advertising noncooperative duopoly equilibria, treating x and y as exogenous. Basic properties of the profit functions generated by these equilibria are shown in Sections IV and V to produce non-standard reaction functions in (x,y) space and to drive the results on entry deterrence described above.

II. Monopoly Equilibrium

Suppose that X has just informed x potential buyers with its initial introductory advertising and subsequent entry is impossible. Then X will optimally maintain its price and quantity constant forever. The present value of its net revenue is

$$\pi(x) = x(1 - p)(p - \gamma)/r = q[(1 - q/x) - \gamma]/r, \tag{3}$$

where r is the relevant discount rate, and p and q are price and quantity, respectively. If q is chosen optimally, price, quantity, and present value are given by

$$p^* = (1 + \gamma)/2,\ q^* = x(1 - p^*) = x(1 - \gamma)/2,\ \pi^m(x)$$
$$= x(1 - \gamma)^2/4r,$$

respectively. It will simplify formulae in what follows to work with re-scaled profits, $\Pi^m(x) = r\pi^m(x)/(1 - \gamma)^2 = x/4$.

Taking into account the cost of introductory advertising, X's present value is equal to $\pi^m(x)$ minus the cost of advertising:

$$w^m(x) = \pi^m(x) - c(x) = x(1 - \gamma)^2/4r + a \ln(1 - x) - f.$$

It will be convenient to define $A = ar/(1 - \gamma)^2$, $F = fr/(1 - \gamma)^2$, and $C(x) = c(x)r/(1 - \gamma)^2$, and to work instead with

$$W^m(x) = \Pi^m(x) - C(x) = x/4 + A\ln(1 - x) - F. \tag{5}$$

Conditional on having prepared to print leaflets, the optimal value of x can also be obtained by maximizing

$$V^m(x) = W^m(x) + F = x/4 + A\ln(1 - x). \tag{6}$$

Differentiation of (5) or (6) establishes that if X elects to enter and subsequent entry is impossible, the optimal fraction informed is given by

$$x^* = M = 1 - 4A. \tag{7}$$

Thus if A exceeds 1/4, it is so expensive to inform potential buyers that firm X will not enter this market even if $F = 0$.[7] We accordingly restrict A to the range $0 < A < 1/4$. In what follows, we will frequently use M instead of A to characterize the variable costs of advertising.

Using the general notation developed above, we next consider how the possibility of subsequent entry affects the optimal choice of x in this model. We proceed as above and consider first the equilibria that emerge after introductory advertising has been completed.

III. Post-Advertising Duopoly Equilibria

Suppose that both x and y have been fixed at some positive values. Since introductory advertising leaflets were sent at random, the total number of buyers informed of either product is given by

$$\theta = 1 - (1 - x)(1 - y) = x + y - xy. \tag{8}$$

Of the informed buyers, $x(1 - y)$ know only of firm X, $y(1 - x)$ know only of Y, and xy are aware of the products of both sellers. This last group of perfectly informed buyers plays a crucial role in the determination of equilibrium. While actual advertising messages are of course not sent totally at random, a late entrant, like Y, will generally reach more buyers who already know of established sellers, like X, the more X has spent to introduce its product. Our task in this section is to find the firm's post-advertising profits as functions of x and y.

There are two natural noncooperative equilibrium concepts that one might employ here: Bertrand (Nash in prices) and Cournot (Nash in quantities). The choice is easy, since Bertrand equilibria do not exist in this model. To see this, consider the optimal choice of Y's price, p , as a function of X's price, p_x. If p_x exceeds the monopoly price, $p^* = (1 + \gamma)/2$, it is optimal for Y to set $p_y = p^*$ and to sell to all y buyers who are aware of it. If p_x is slightly below the monopoly price, Y does better by undercutting X slightly and selling to the same set of buyers. Finally, if p_x is low enough, Y is better off selling to the $y(1 - x)$ buyers who are aware of it but not of X at the monopoly price than it is undercutting X. The condition for this is

$$(1 - \gamma)^2 y(1 - x)/4r \geq y(1 - p_x)(p_x - \gamma)/r,$$

$$\text{or } p_x < [(1 + \gamma) - (1 - \gamma)\sqrt{x}]/2.$$

Symmetric reasoning yields the optimal p_x as a function of p_y, and a sketch of these reaction functions makes clear the non-existence of Bertrand equilibrium.

We therefore assume that the post-advertising duopoly equilibrium would be Cournot and that this is known in advance by both firms. There are two types of Cournot equilibria in this model, depending on whether or not both firms make sales to the xy perfectly informed buyers.

If both firms sell to these buyers, the situation is exactly equivalent on the margin to one in which both firms have informed all θ knowledgeable buyers. The perfectly informed buyers ensure that all buyers pay the same price, and price and aggregate quantity must satisfy eq. (1). Thus as long as both firms make positive sales to perfectly informed buyers, X's profit function is

$$\pi^X = q_x\{[1 - (q_x + q_y)/\theta] - \gamma\}/r.$$

Firm X's Cournot reaction function is just

$$q_x = [\theta(1 - \gamma) - q_y]/2. \tag{9}$$

Firm Y's reaction function for this region simply interchanges the subscripts, and the unique solution to this pair of linear equations is

$$q_x = q_y = \theta(1 - \gamma)/3. \tag{10}$$

In order for this to describe an equilibrium, it must be possible for q_x to equal q_y and for both firms to make some sales to perfectly informed buyers, as hypothesized. This requires that at least half the informed set know of X and that at least half know of Y. That is, we must have $x \geq \theta/2$ and $y \geq \theta/2$, or, using equation (8),

$$y/(1 + y) \leq x \leq y/(1 - y). \tag{11}$$

Now suppose that only one firm makes sales to the perfectly informed buyers. (As long as at least one firm receives a price less than unity for its output, at least one firm must sell to these buyers.) This means that one of the inequalities in (11) must be violated, since (10) gives the unique equilibrium if both hold. Suppose that the right-most inequality is violated, so that y is smaller than x. Equilibrium then involves Y selling to all y buyers who are aware of it, with X selling to the $x(1 - y)$ who know *only* of it. Thus only Y sells to the perfectly informed buyers. At such a point, any increases in q_x would involve some sales to these individuals, so that (9) must hold at equilibrium. As it is easy to show that there can be only one price in a Cournot equilibrium,[8] the other condition equates prices received by the two firms:

$$p_x = 1 - q_x/[x(1 - y)] = p_y = 1 - q_y/y. \qquad (12)$$

One can show directly that at the unique solution to (9) and (12), Y will neither wish to decrease its output (and allow X to make sales to perfectly informed buyers) nor to increase its output (and receive less per unit than X does). Firm Y's demand is in fact kinked at the equilibrium point; it is more elastic for quantity decreases than for increases.

Solving for the profits earned at these equilibria, and multiplying by $r/(1 - \gamma)^2$ as above, we obtain the re-scaled profit function for post-advertising Cournot duopoly equilibrium:

$$\Pi^x(x,y) = \begin{cases} \theta xy(1 - x)/(2\theta - x)^2, & 0 \leq x \leq y/(1 + y): \text{ Region A} \\ \theta/9, & y/(1 + y) \leq x \leq y/(1 - y): \text{ Region B} \\ \theta x^2(1 - y)^2/(2\theta - y)^2, & y/(1 - y) \leq x \leq 1: \text{ Region C} \end{cases} \qquad (13)$$

By symmetry, Y's re-scaled profit, Π^y, is given by this same function with arguments transposed. The analysis of optimal advertising levels in the next two sections rests on this function. Define W^x, W^y, V^x, and V^y just as in (5) and (6).

The three Regions defined by (13) are important in what follows. If x is in Region B, so is y. If x is in Region A, y is in Region C. (That is, $x \leq y/(1 + y)$ implies $y \geq x/(1 - x)$.) The function $\Pi^x(x,y)$ is continuous in x for fixed y, but its derivatives are discontinuous at the boundaries between Regions.

In general, if $x > y$, then $\Pi^x/\Pi^y < x/y$. These equilibria thus exhibit disadvantages of size. If x is in Region B, so that y must be also, the two firms split the market even if $x \neq y$. The fact that one firm has a larger number of informed buyers than the other has no effect on the Cournot equilibrium as long as the marginal buyers are perfectly informed and price discrimination is ruled out. Increases in either x or y would simply increase the total market to be divided and thus benefit the other firm. If $x > y$, $c(x) > c(y)$, so that if both firms are in Region B, $W^x - W^y = V^x - V^y \geq 0$; firm X has a lower present value net of advertising costs. At equilibrium, the number of X's customers unaware of Y exceeds the number of Y's customers unaware of X, but without the ability to discriminate, this does X no good.

If x is in Region C, so that y is smaller and in Region A, one can show that market price is a decreasing function of y/θ. The smaller the set of buyers aware of Y, the milder is X's reaction to its entry.[9] And the larger is x, the smaller is y/r for any y.

Figure 1 shows the average return on informed buyers, $\Pi^x(x,y)/x$, for several values of y. (By symmetry, this graph also shows $\Pi^y(x,y)/y$ for the same values of x.) The kink in the curves for y = 0.1 and 0.3 occurs as X passes from Region B to Region C. The locus of those kinks, which forms a lower envelope for average returns, is given by $\Pi^x(x,y)/x = 2/[9(1 + y)]$. If the re-scaled average cost function for informed buyers, $C(x)/x$, is anywhere below this envelope, entry deterrence is clearly impossible.

*** INSERT FIGURE 1 NEAR HERE ***

As intuition suggests, X is best off when $y = 0$. Then $\Pi^X/x = 1/4$, but $\Pi^X/x < 1/4$ for all positive x if y is positive. But, as Figure 1 shows, if y is positive, increases in y serve to raise X's average return in Regions A and B. (Formally, Π^X_y is positive for x in Region A or Region B.[10]) Such increases enhance the advantages of small scale (relative to Y) that X can enjoy in those Regions. Figure 2 suggests that for at least some average cost functions, $C(x)/x$, entry deterrence might involve advertising less than would otherwise be optimal in order to keep average returns below average costs in those Regions. To see if this suggestion is correct, we must employ (13) in an explicit analysis of duopoly choices of x and y.

IV. Advertising Reaction Functions

Suppose that both X and Y have designed leaflets and prepared to print them, so that we can ignore their fixed and sunk costs, F. In this section we also ignore the actual sequence in which x and y are chosen. Sunk costs and timing considerations are brought back into the analysis in Section VI. Under these assumptions, we first investigate X's optimal response, $x^*(y)$, to exogenously fixed values of y. (By symmetry, Y's optimal response to X, $y^*(x)$, is given by the identical function.) We then look for Nash equilibria in real advertising levels and describe the behavior of profits along reaction functions.

The usual necessary condition for the optimality of x^* is

$$\Pi^X_x(x^*,y) - C_x(x^*) = \Pi^X_x(x^*,y) - A/(1 - x^*) = 0. \qquad (14)$$

One can show that $C_{xx} > 0$, $\Pi^X_{xx} < 0$ in Regions A and C, and $\Pi^X_{xx} = 0$ in Region B, so that the second-order condition for a maximum is satisfied at a solution to (14). Because $\Pi^X_x(x,y)$ is discontinuous in x at the boundaries between Regions, however, we cannot simply differentiate (14) to explore the properties of $x^*(y)$.

Figure 2 depicts the geometry of the situation. One can show that Π^X_{xy} is positive in Region A and negative in Regions B and C, so that the shifts in Π^X_x shown in Figure 2 are qualitatively valid everywhere. The limiting values of Π^X_x at Regional boundaries are given in Figure 2 and used in what follows. (This derivative equals $(1 - y)/9$ throughout Region B.)

*** INSERT FIGURE 2 NEAR HERE ***

If marginal costs of x are given by $C^a_x(x)$, Figure 2 shows that x^* is in Region C if $y = 0.25$, and x^* is in Region B if $y = 0.35$. It should be clear from the geometry of the Figure that the transition between these two regions involves a discontinuous drop in x^*. For this cost function, there will exist some value of y, call it D, such that (14) has two solutions, one in Region B and one in Region C, that yield equal values of V. As y is increased beyond D, x^* drops discontinuously from Region C (where it exceeds $y/(1 - y)$) to Region B (where it is less than $y/(1 - y)$). If the C_x schedule were flatter than C^a_x, X's transition from Region C might involve a discontinuous drop to Region A or to the boundary between A and B. As a computational matter, if C_x intersects Π^X_x twice (treating Π^X_x as having a vertical segment at the A/B boundary), one must compare V^X at the two intersections in order to determine x^*.

In contrast, as long as C_x slopes upward, the discontinuity in

Π^x_x at the A/B boundary does not induce a discontinuity in $x^*(y)$. With marginal cost schedule C^b_x in Figure 2, for instance, equation (14) does not have a solution, but x^* is clearly equal to $y/(1 + y)$ for both values of y shown. Since $\Pi^x(x,y)$ is continuous, one can use the Region B portion of (13) to compute X's present value.

Except when x^* is on the A/B boundary, it is determined as a solution to (13), and the slope of X's reaction function is thus given by

$$dx^*/dy = - \Pi^x_{xy}/[\Pi^x_{xx} - C_{xx}].$$

This has the sign of Π_{xy} by the second-order condition. Given the sign pattern of this derivative (as noted in the paragraph just below equation (14)), it follows that dx^*/dy is positive in Region A and negative in Regions B and C. If x^* is on the A/B boundary, as in Figure 2 with marginal cost schedule C^b_x, it is clear that dx^*/dy is positive.

We are now able to describe the qualitative properties of $x^*(y)$ for any upward sloping C_x schedule. If $y = 0$, $x^* = M$ by definition. As y increases from zero, x^* initially declines smoothly in Region C. As y increases beyond a critical value, D, x^* drops discontinuously to a value below $y/(1 - y)$ and thus out of Region C. If this drop yields x^* above $y/(1 + y)$, and thus in Region B, further increases in y lower x^* continuously until $y = 1$ or the A/B boundary is encountered at $x^* = y/(1 + y)$. (For the C(x) function assumed here, $x^*(1)$ is always in Region A or on the A/B boundary.[11]) If the A/B boundary is encountered for $y < 1$, x^* increases in y thereafter. If the discontinuous transition from Region C drops x^* to the A/B boundary or into Region A, all further increases in y cause x^* to increase.

Figure 3 shows that the pair of reaction functions, $x^*(y)$ and $y^*(x)$, for the particular cost function assumed here with $M = 0.5$.[12] The discontinuity at D involves a drop to the A/B boundary in this case; it is never optimal to operate in Region B. For large y, x*(y) moves from the A/B boundary into the interior of Region A.

*** INSERT FIGURE 3 NEAR HERE ***

Let us next consider Nash equilibria in this model. No such equilibrium in x and y exists in Figure 2. In general, one can show that Nash equilibria must involve both firms operating in Region B.[13] A necessary condition for (x,y) to be a Nash equilibrium is thus that both firms' first-order conditions for that Region be satisfied:

$$(1 - y)/9 = A/(1 - x), \quad \text{and} \quad (1 - x)/9 = A/(1 - y). \tag{15}$$

These conditions are identical, however, so that in general if there is one Nash equilibrium there will be a continuum of such equilibria.[14] There exist x and y between zero and one satisfying (15) as long as $A < 1/9$, or $M > 5/9$. But this is only a necessary condition; the necessary and sufficient condition requires both firms to be in Region B, so that $x = x^*(y)$ and $y = y^*(x)$. By the symmetry of the situation, there exist equilibria if and only if the symmetric point $x = y = 1 - \sqrt{9A}$ is an equilibrium. Numerical evaluation of reactions functions corresponding to different values of M indicates that this condition is satisfied for $M \geq E$, where E is between 0.82 and 0.83. Figure 4 shows the reaction functions for $M = 0.86$. All points $[x^*(y),y]$ for $D < y < T_1$ are Nash equilibria, as indicated. For this value of M, it is never optimal to operate in Region A, and the discontinuity at D leads to operation in Region B.

*** INSERT FIGURE 4 NEAR HERE ***

Finally, let us see how V varies along the reaction function. One can show that Π_x^x is negative in Region C and positive in Regions A and B. Thus by the envelope theorem, for any upward sloping C_x schedule, V^x is reduced by increases in y for y < D, minimized at y = D, and increased by increases in y for y > D. Further, D is always less than M, as in Figures 3 and 4. Since reaction functions are negatively sloped in Region C, J < M, where J is the limiting value of x*(y) as y approaches D from below. (See Figures 3 and 4.) Since the point x = J, y = D is in X's Region C, it must be that J > D/(1 - D) > D, so that M also exceeds D as asserted.

V. Advertising by the First Entrant

We are now in a position to analyze the optimal choice of x by firm X, the first entrant, on the assumption that it knows that Y will appear after x has been chosen (and advertising leaflets have been sent to potential buyers) and contemplate entry. The results depend on what is assumed about X's options after Y's entry; we consider two cases. In the first case, X can credibly commit not to increase x after Y's entry, so that x will in fact never change. (Since buyers never forget, decreases in x or y are not possible, just as decreases in production capacity are usually ruled out in the corresponding literature.[15]) Under some conditions, destruction of the materials necessary to print more leaflets may serve to accomplish this. In the second case, X cannot make such commitments and is restricted to credible x's, defined as those values of x such that X will not wish to increase advertising if Y enters on its reaction function. Formally, x_0 is credible if and only if

$$x^*[y^*(x_0)] \leq x_0 \qquad (16)$$

In Figure 3, x is credible unless it is less than T. In Figure 4, there are two ranges of credible x's: $D \leq x \leq T_1$ and $x \geq T_2$. If X selects a credible x, Y's best response if it enters is y*(x), and the game is over. If X cannot commit in advance not to increase x and it selects an x that is not credible, the situation becomes quite complex. Both X and Y must analyze possible sequences of increases in x and y over time. Limiting X to credible advertising levels avoids the necessity of solving such dynamic games, but it may over-state the effects of X's inability to commit not to increase advertising after Y's entry.

If F is large enough, entry will be blockaded in the sense of Bain (1956) in both cases. That is, if X sets x = M, as if entry were impossible, and $V^y[M,y^*(M)]$ is less than F, Y's entry will be deterred. (The point x = M is always credible.[16]) The upper boundary in Figure 5 is derived by setting F equal to this critical value and scaling by X's present value net of leaflet preparation costs. (If $F \geq V^m(M)$, the market is not profitable for X even if it is free from the threat of entry.)

*** INSERT FIGURE 5 NEAR HERE ***

If entry is not blockaded, it is necessary to make some assumption about X's post-entry options. Let us deal first with the simple case in which the x it selects is never changed. Given the threat of Y's entry, X can either select the Stackelberg value of x, which maximizes $V^x[x, y^*(x)]$ in anticipation of Y's arrival, or it can attempt to deter Y's entry. Since $V^y[x, y^*(x)]$ is minimized at x = D and increases in x for x > D, and since D < M, it is clear that X will never set x > M in an attempt to deter entry. Such over-investment in introductory advertising would make entry more attractive to Y, not less attractive. Entry deterrence thus must involve setting x between D and M. If $F < V^{y}[D, y^{*}(D)]$, deterrence is impossible, since the minimum value of W^{y} that X can impose is non-negative. The lowest curve in Figure 5 presents a numerical evaluation of this critical level of F, scaled as before.[17]

If F is between the upper and lower critical levels, so that deterrence is neither unnecessary nor impossible, X's optimal strategy is found by comparing the profits yielded by optimal deterrence with those obtained at Stackelberg equilibrium (with X as leader). One can show that $V^{m}(x)$ is concave, so that the most profitable deterrence strategy involves setting x as close to M as possible. This is accomplished by choosing the unique x between D and M such that $F = V^{y}[x, y^{*}(x)]$. At this point, (6) gives X's present value. The Stackelberg return, to which this must be compared, is found by maximizing $V^{x}[x, y^{*}(x)]$. In fairly extensive computations, deterrence was always more profitable than Stackelberg equilibrium when F was between the upper and lower critical levels.[18] It is thus apparently always optimal in this model to deter entry when deterrence is both possible and necessary and x can commit never to increase its advertising after Y's entry. But, in sharp contrast to models involving investment in production capacity, deterrence is here accomplished by under-investment relative to the monopoly level, not over-investment.

If deterrence is impossible, and apparently only then, X's best policy is to act as a Stackelberg leader in anticipation of Y's entry. For all values of $M \geq 0.06$ for which computations were performed, this policy called for X to select the smallest x less than D at which the relevant functions were evaluated.[19] Thus Y's post-entry profits are apparently generally minimized at X's Stackelberg point. While y is indifferent between x's just above and just below D, X prefers the latter because they induce sharply larger values of y from which it benefits. At least for $M \geq 0.06$, and possibly more generally, y exceeds x at X's Stackelberg point, as Figures 3 and 4 suggest. In this case, X's position as first mover enables it to capture advantages associated with being small relative to its rival; V^x exceeds V^y at all Stackelberg points computed.

The results change in interesting ways when X is restricted to credible x's. In order for deterrence to be possible in this case, F must exceed the minimum value of $V^y[x, y*(x)]$ over all credible x's. In Figure 3, x's below T are not credible, so that if F is less than $V^y[T, y*(T)]$, it is not possible to deter entry credibly. In Figure 4, however, points arbitrarily close to D are credible, and the credibility constraint does not reduce the set of F's for which deterrence is possible. The middle curve in Figure 5 is a numerical evaluation of the upper bound of the region in which credible deterrence is impossible.[20] Note that at M = E, where Nash equilibria in x and y come into existence and the geometry changes from that of Figure 3 to that of Figure 4, this curve drops discontinuously to the lower curve, with which it coincides for all M > E.

If deterrence is neither impossible nor unnecessary, we must again compare deterrence profits with those obtained by Stackelberg leadership. The Stackelberg return is found by maximizing $V^x[x, y*(x)]$ over credible values of x. In all cases for which computations were performed, $V^m(x)$ exceeded this return for all credible x's between D and M.[21] Thus, even if X is limited to credible values of x, it is apparently optimal to deter entry whenever possible. As before, deterrence must always involve advertising less than a protected monopolist would.

Finally, if deterrence is impossible, and, again, apparently only then, X's optimal policy is Stackelberg leadership. The credibility constraint (16) is apparently always binding: none of the unconstrained Stackelberg points computed satisfied it. In this case, $V^x[x, y^*(x)]$ was always maximized at the smallest credible x examined.[22] For $M < E$, when no Nash equilibria exist, X thus apparently optimally chooses a point like T in Figure 3 if Y's entry is inevitable. As that Figure suggests, $y^*(x)$ was less than x at all such points examined, and V^y exceeded V^x. Since x cannot be credibly set below $y^*(x)$ for $M < E$, the credibility constraint apparently makes it better to enter second than first! For $M > E$, values of x arbitrarily close to E become credible, as in Figure 4, and computations indicate that X optimally selects the Nash equilibrium with the smallest possible value of x.[23] At such points V^x exceeds V^y, though $y^*(x)$ exceeds x. When Nash equilibria exist, there is thus apparently still an advantage in entering first even though later entry cannot be deterred. As before, that advantage turns on the ability to be small relative to the later entrant.

VI. Conclusions

The assumptions stated in Section I were chosen in an attempt to produce a simple model resembling as closely as possible models in which over-investment in production capacity may credibly deter entry, while allowing advertising to have plausible effects on buyers' behavior.[24] The resulting model turned out to be of surprising (to me, at least) complexity and to differ in basic ways from the models after which it was patterned. Here, as in those models, durability and scale economies can interact to produce blockaded entry. But if entry is not blockaded, optimal deterrence here involves under-investment, whereas deterrence strategies involving production capacity employ over-investment.

Moreover, this model exhibits pervasive disadvantages of size that do not have analogs in most models not involving advertising.

The analysis presented here suggests that the strategic implications of investments in at least some kinds of advertising may differ dramatically from those of investments in tangible, productive assets. Two sources of such differences come to mind. First, investment in productive assets affects the relation between cost and units of output, while investment in advertising fundamentally acts on individuals' demand curves. When individual buyers can vary the number of units they purchase, as in this model, this formal difference may have basic strategic implications. Second, investment in productive capacity generally discourages entry by making lower prices more attractive to the incumbent. But investment in advertising that provides the incumbent with a set of loyal customers that would not find an entrant's wares attractive (perhaps, as here, because they would be unaware of the entrant's existence) may make it less eager to cut price in response to entry and give up profits that could be earned from those loyal customers. With price discrimination ruled out, as it is here, an incumbent's investment in advertising may make entry more attractive by guaranteeing the entrant a friendlier welcome.[25] Both these factors are present in this model. The second is clearly visible in the post-advertising equilibria of Section III, and it appears to drive the main results obtained here.

I do not claim that the model presented in this essay provides a useful description of most markets or indeed of any actual market. While I think its assumptions are natural and have some plausibility, its results run counter to the intuitions of many economists and to some recent research in related areas.[26] More work is clearly required to see exactly what features of this model drive those results and to delineate the conditions under which over-advertising can optimally and credibly deter entry.

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Footnotes

I am indebted to the National Science Foundation for financial support, to Severin Borenstein for excellent research assistance, and to participants in a seminar at the University of Toronto for valuable comments on an earlier version of this essay. The usual waiver of liability applies.

1. See, for instance, Scherer (1980, pp. 246-8).
2. Dixit (1982) has recently provided a clear summary of this work. He also discusses complementary work on "reputation effects," which are ignored here.
3. The seminal paper here is Spence (1977), which focuses on durable investment but does not deal with credibility. The result stated in the text seems due to Dixit (1980); see also Schmalensee (1981) and Dixit (1982). The importance of investment durability is made clear by Eaton and Lipsey (1981).
4 There is a good deal of controversy about both durability and scale economies; see Comanor and Wilson (1979).
5. Spence (1977, 1980) has dealt with advertising in this context, but his models suffer from the same basic problem as the Bain/Sylos analysis.
6. Baumol and Willig (1981) have recently stressed the conceptual importance of the distinction between fixed and sunk costs.
7. One can show that a necessary and sufficient condition for a monopoly secure against entry to have a positive W^m is that $c(x)/x$ attain its minimum at $x < M$.
8. Suppose the contrary, and assume that equilibrium involves q's such that $p_x > p_y$. Then X must sell only to the $x(1 - y)$ buyers who

know only of it, and Y sells to all y who are aware of it. It is then clearly optimal for X to obtain the monopoly price, p*, from its captive audience by producing the corresponsing monopoly quantity, $x(1 - y)(1 - c)/2$. But Y can raise its price (by lowering its output) and increase its profits until it too obtains the monopoly price from its customers.

9. This is intimately related to the "judo economics" of Gelman and Salop (1982). They find that a potential entrant that can credibly limit its output may be able to profitably enter against an established monopoly despite demand and cost disadvantages.

10. Subscripts are used to denote partial derivatives here and in all that follows.

11. Use of the limiting values of Π^X shown in Figure 2 establishes that sufficient conditions for x* not to lie in B or C are (a) $A/[1 - y/(1 + y] > (1 - y)/9$, and (b) either $y \geq 1/2$ or $A/[1 - y/(1 - y)] \geq 7(1 - y)/27$. The first of these is satisfied for $y > (1 - 9A)/(1 + 9A)$, and the second is satisfied for $y > (7 - 27A)/14$. Both of these critical values are always less than one. So if y is close enough to unity, x*(y) must be either in Region A or on the A/B boundary.

12. The profit function in (13) is such that points on the reaction function cannot generally be obtained analytically. Proceeding as in footnote 11, the program employed first ascertains the Regions in which solutions to (14) exist, and it checks to see if x* may be on the A/B boundary. It then computes solutions to (14). In Regions A and C, this requires numerical evaluation of the relevant roots of fourth degree polynomials. A direct comparison of profits is used to determine x* if multiple candidate values are thus identified.

13. There can be no equilibria with either firm on the A/B boundary, since this would imply that the other was on the B/C boundary, and it is never optimal to operate there. Suppose an equilibrium exists with x in Region A and y in Region C. Then if $z = x/(1 - x)$, equilibrium involves $z < y$. Writing down the corresponding first-order conditions and solving both for $A(1 + z)(2y + z)^2/y$, one finds that an equilibrium of this sort,

$$T(z;y) = z^3 + z^2[y(1 - y)] + z[3y^2(1 - y) - 3y] + 2y^2[y(1 - y) - 1] = 0.$$

To show that no equilibria outside Region B exist, it suffices to show that this equation, treated as a cubic in z for given y, has no real roots between zero and y. Since the maximum value of $T(y,y)$ is negative, T is always negative when $z = y$. Since T must be positive for large z, $T(z;y) = 0$ thus has one real root larger than y. At $z = 0$, T is negative and decreasing in z, it attains a local maximum at some negative z, and it is clearly negative for large negative values of z. It thus follows that $T(z;y) = 0$ has either a pair of negative roots or a pair of complex roots. Since it has no real roots between zero and y, the proof is complete.

14. This is a property of the particular $C(x)$ function assumed here. If $C(x) = F + Ax/(1 - x)$, which has roughly the same shape, there is at most one Nash equilibrium for any value of A. For this cost function, numerical evaluation of the reaction function indicates that such equilibria exist for all $A \leq 0.035$, which corresponds to $M \geq 0.626$.

15. See the references cited in footnote 3, above.

16. By the shapes of reaction functions in this model, if $x^*(1) \leq M$, it follows that $x = M$ is credible, since X will never want to raise x no matter what Y does. Footnote 11 establishes that $x^*(1)$ is either in Region A, with $x^* < y/(1 + y) = 1/2$, or on the A/B boundary with $x^* = 1/2$. Referring to Figure 2, x^* is on the boundary if and only if $(5 - y)/27 \geq A/[1 - y/(1 + y)]$, or $A \leq 2/27$. But $A \leq 2/27$ implies $M = 1 - 4A \geq 19/27 > 1/2$. Thus $x^*(1) < M$ for $A \leq 2/27$. For all larger A, X's Region A first-order condition when $y = 1$ is

$$(2 + 3z - z^3)/(2 + z)^3 = A(1 + z),$$

where $0 \leq z = x/(1 - x) \leq 1$. At $z = 0$, the left-hand side of this equation equals 1/4, while the right-hand side equals A, which is less than 1/4. The left-hand side is decreasing in z, so that $A(1 + z) < 1/4$ at a solution. Substituting for z, this last inequality establishes that $x^*(1) < M$ for $A > 2.27$ as well, and the proof is complete. (In Figure 3, $A > 2/27$, while $A < 2/27$ in Figure 4.)

17. For $M = 0.02, 0.04, \ldots, 0.98$, we searched over $x = 0.002, 0.004, \ldots, 0.998$ to find the minimum value of $V^y[x, y^*(x)]$. Since the true minimum was thus never found exactly, the lowest curve in Figure 5 is too high by an unknown but probably small amount. For small M the search over x involves a coarse grid in the relevant region, and the results are correspondingly less reliable. (Indeed, some model properties known to hold globally were violated in the output for $M = 0.02$ and $M = 0.04$.) While some of the general results suggested by these computations and described in what follows may be susceptable to analytical proof, this model is both so special and so algebraically complex that the slight strength this would add to our

conclusions hardly seems worth the substantial effort that would be required.

18. See footnote 17. The Stackelberg profit was taken to be the largest value of $V^x[x, y^*(x)]$ encountered, and this was compared to $V^m(x)$ for all x's examined between D and M.

19. See footnote 17. Results for M = 0.02 and 0.04 are suspect for the reasons noted there. The finding reported in the text suggests that X's Stackelberg point is strictly undefined, as there is no "largest x less than D".

20. The procedure described in footnote 17 was used, except that the search was restricted to values of x passing the credibility test, given by inequality (16) in the text.

21. See footnote 17.

22. See footnote 19.

23. Again, this means that X's Stackelberg point may be undefined, as there is in general no "Nash equilibrium with the smallest x".

24. Baldani and Masson (1981) allow the incumbent monopolist to invest in "goodwill," which determines post-entry share but does not affect pre-entry sales. It is not clear what assumptions about buyer behavior would produce such a demand structure, though aggregate structures involving "goodwill" are commonly assumed.

25. See footnote 9.

26. In particular, these results seem at odds with recent theoretical and empirical work that points to general advantages enjoyed by pioneering brands; see Schmalensee (1982) and the references there cited. This comparison suggests the importance of the assumption here that advertising removes all uncertainty about product quality.

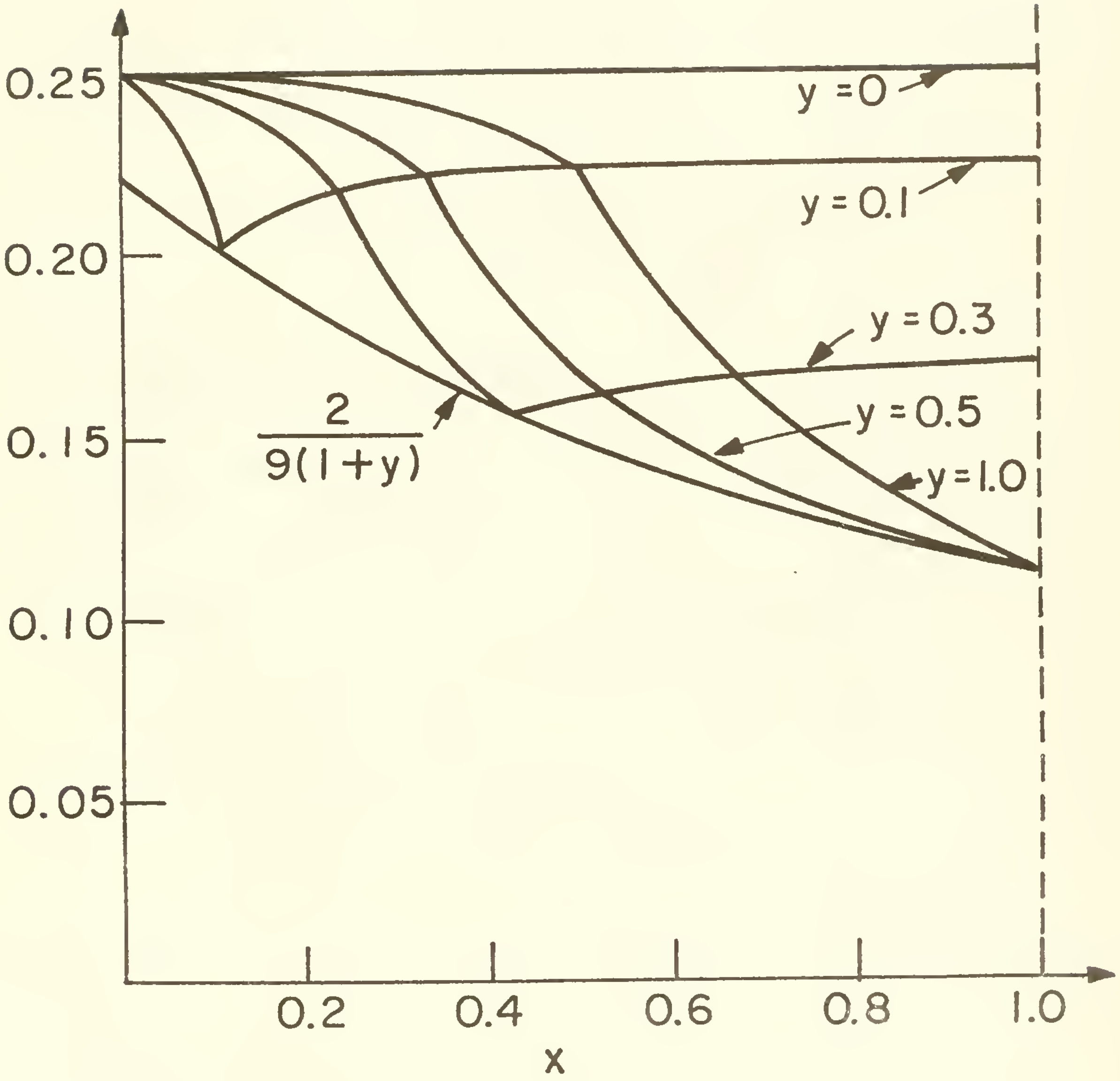
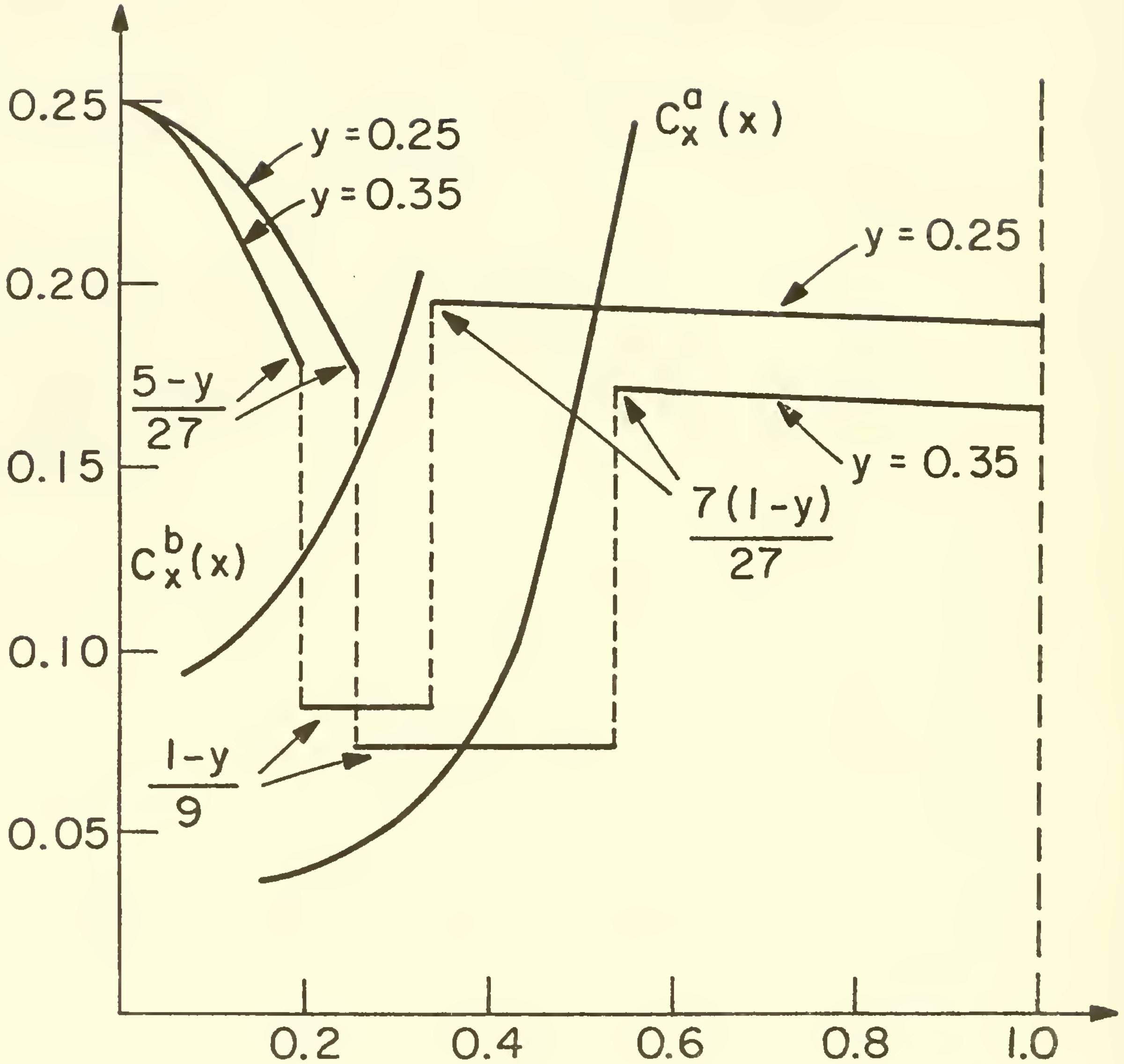


Fig. 1 $\Pi^{x}(x,y)/x$


0.25
0.20
0.15
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0.8
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x
y = 0.25
y = 0.35
$C_x^a(x)$
y = 0.25
$\frac{5-y}{27}$
y = 0.35
$\frac{7(1-y)}{27}$
$C_x^b(x)$
$\frac{1-y}{9}$

Fig. 2 $\Pi_x^x(x,y)$

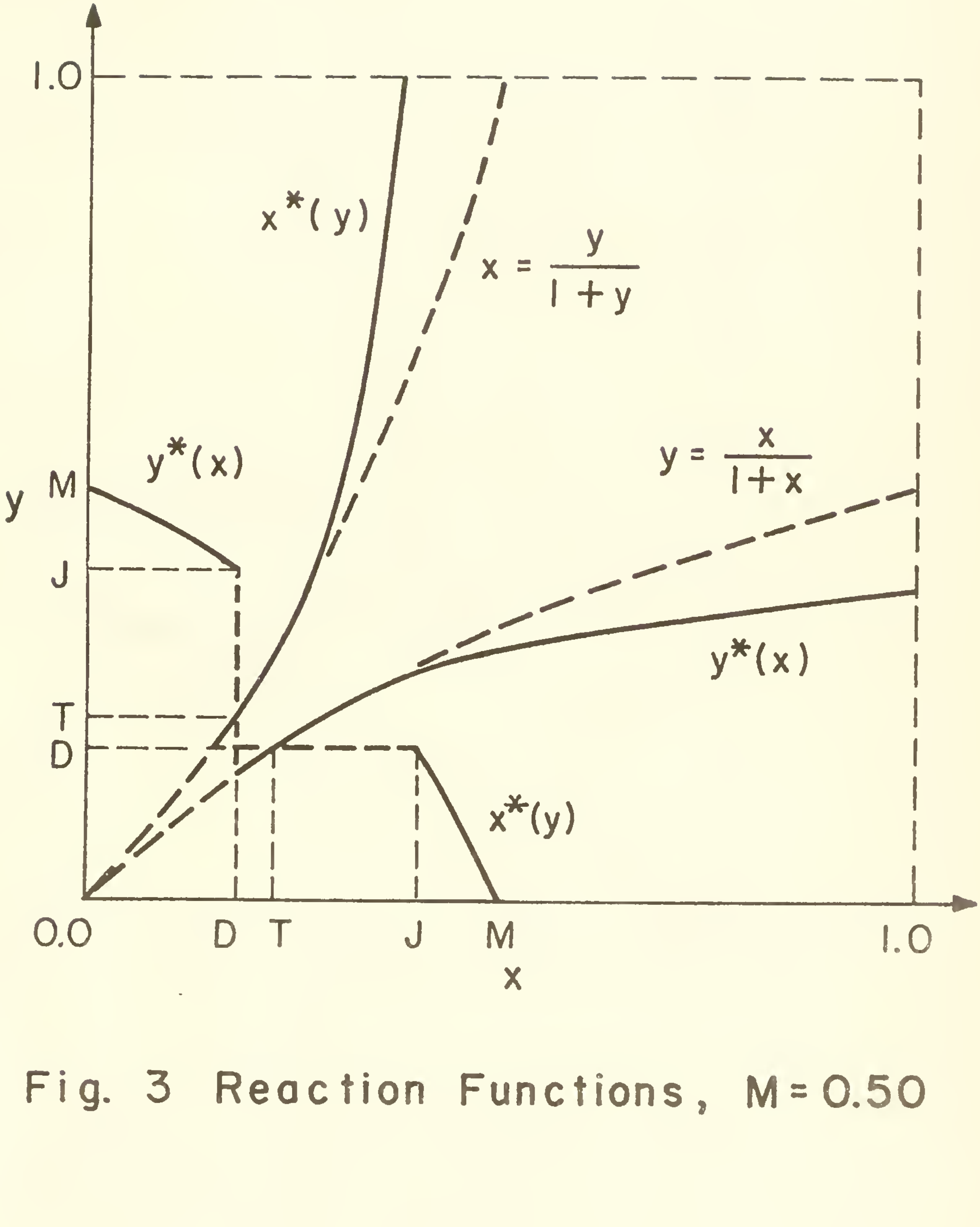

Fig. 3 Reaction Functions, M=0.50

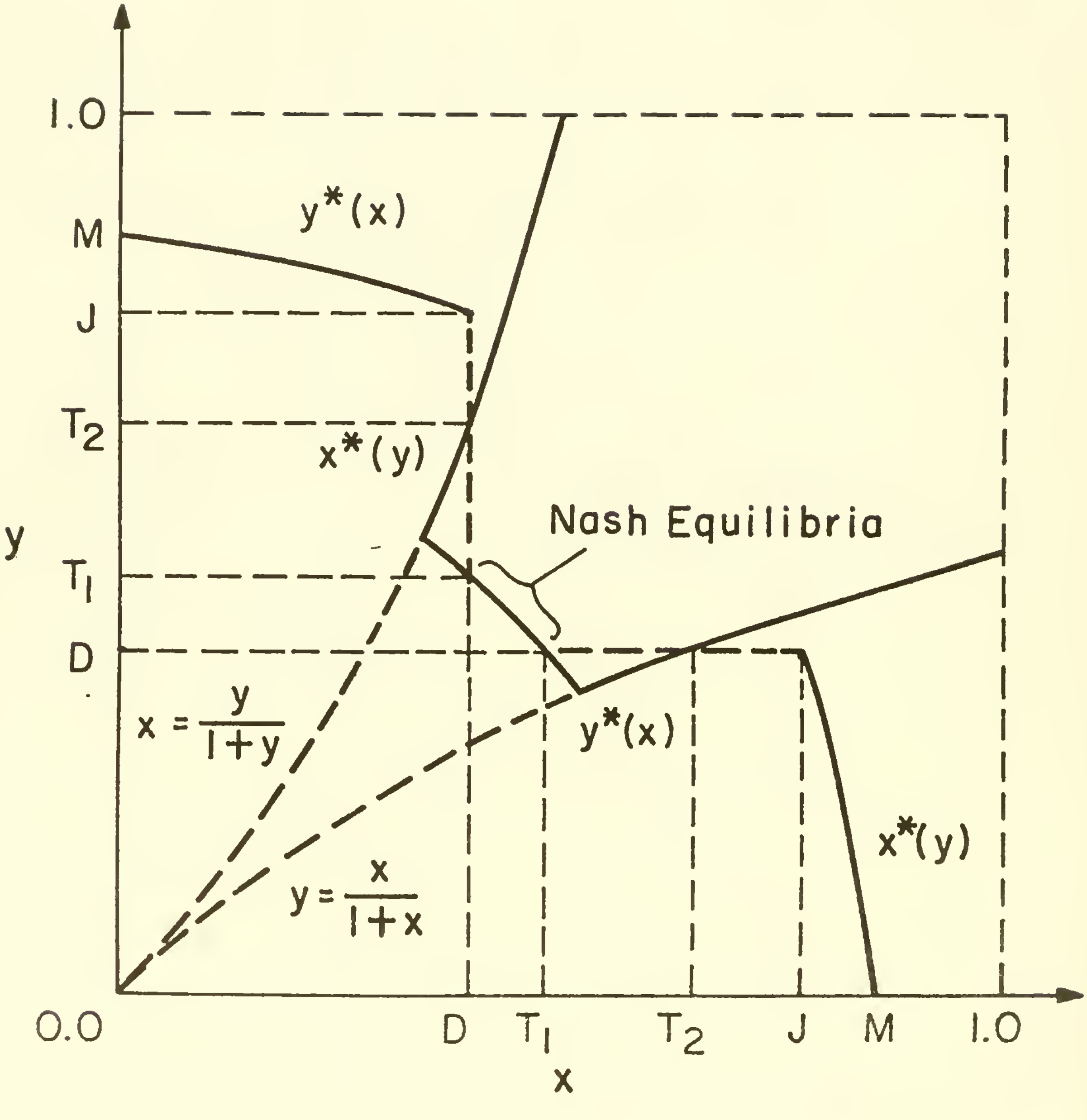

Fig. 4 Reaction Functions, M = 0.86

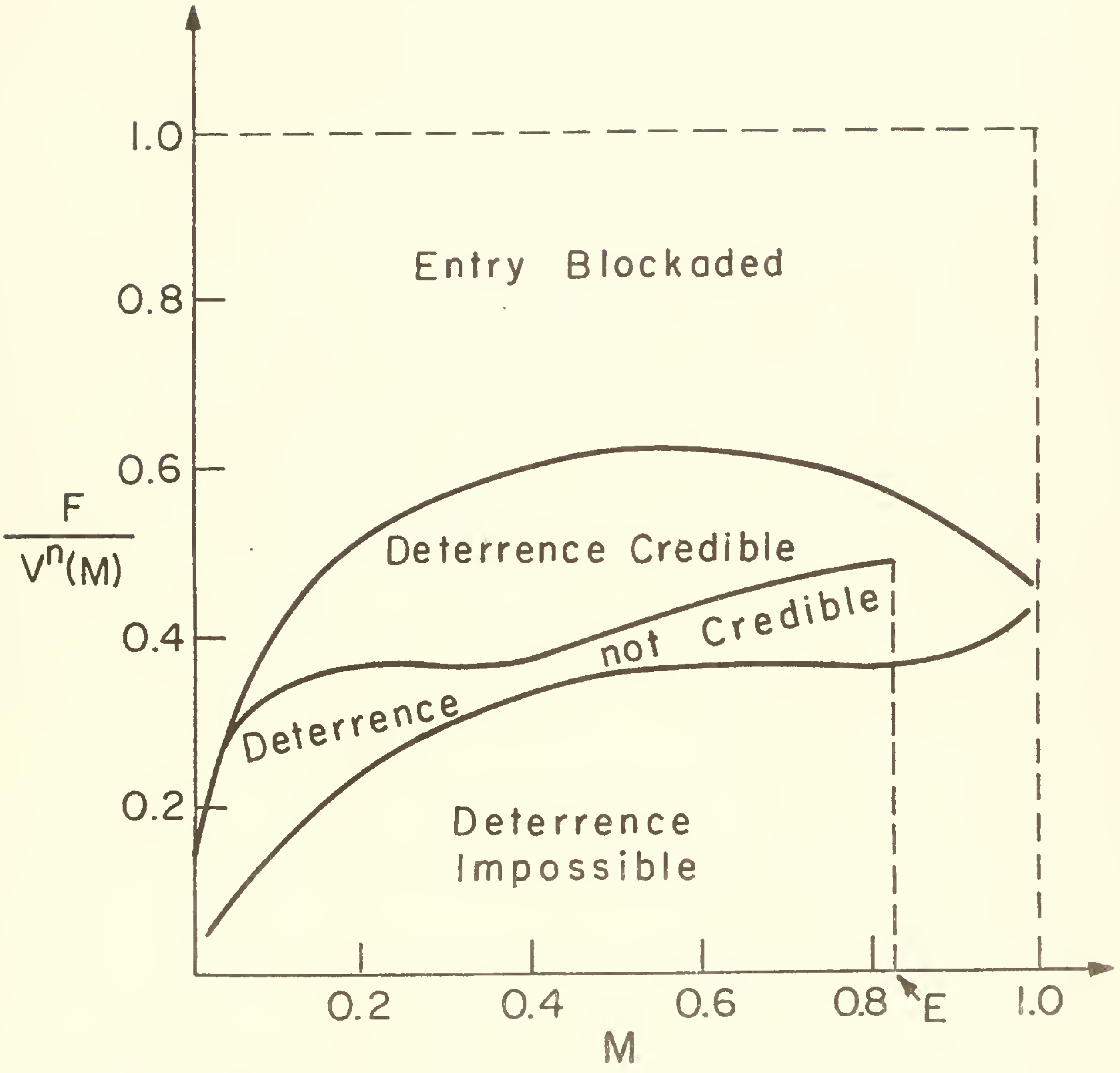

Fig. 5 Conditions of Entry

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